We know from before that

$$e^{j\theta} = \frac{A+D}{2} + j\left[1-\left(\frac{A+D}{2}\right)^2\right]^{1/2} = \cos\theta + j\sin\theta.$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{A+D}{2}\right) = \cos^{-1}\left(1-\frac{d}{f_2}\right).$$

$$f_2 = \frac{2}{3}d \quad \Rightarrow \quad \theta = \cos^{-1}\left(1-\frac{3}{2}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}\ (120°).$$

∴ our solution for the position r is given by

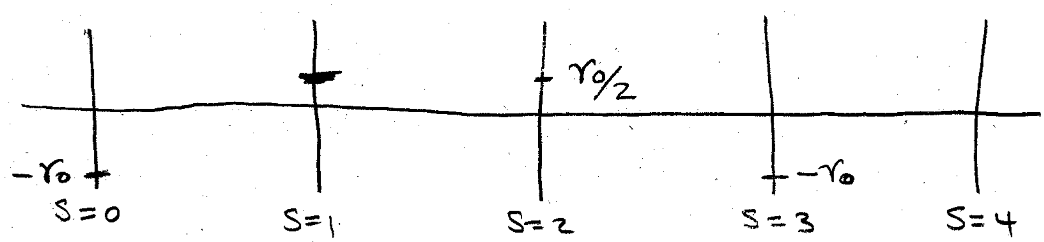
$$r = r_{max}\sin\left(s\frac{2\pi}{3}+\alpha\right)$$

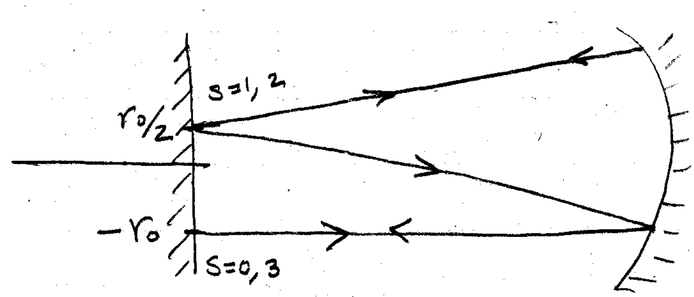
at $s=0$ $\quad r_{max}\sin\alpha = -r_0 =$

$$\sin\alpha = -1. \qquad r_{max} = r_0.$$

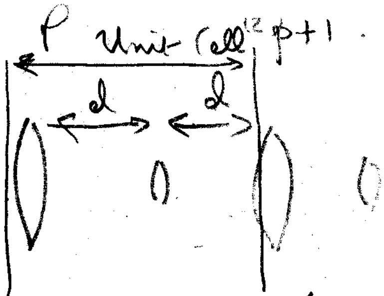
$$\alpha = -\frac{\pi}{2}$$

$$r = r_0\sin\left(s\frac{2\pi}{3}-\frac{\pi}{2}\right). \qquad (\text{location on } M_1)$$





If we want to know location at other mirror M_2 we can use a different unit cell.



Solution $r = 2r_0 \sin\left(p\frac{2\pi}{3} - \frac{\pi}{6}\right)$.

Different choices of unit cell $\Rightarrow$ different solutions and information only about the position at the edge of that unit cell.

Repetitive Ray Paths.

– How many round trips before the ray returns to its original position?

$$r_s = r_{max}(s\theta + \alpha).$$

$$\theta = \cos^{-1}\left(\frac{A+D}{2}\right)$$

For repetition m rounds trips is required where

$$m\theta = 2n\pi \quad \text{– some multiple of } 2\pi.$$

$$n < \frac{m}{2}.$$

$$m = \frac{2\pi n}{\theta} \quad \text{for } \theta = \frac{2\pi}{3} \qquad m = 3n.$$

$$n < \frac{m}{2}.$$

$n = 1,\ m = 3$ is a solution.

Initial Conditions — Stable cavity.

If cavity is stable $\theta = \cos^{-1}\left(\frac{A+D}{2}\right)$. $\cos\theta = \left(\frac{A+D}{2}\right)$

Suppose initial conditions are r_0 and r_0'

$$r = |r_{max}| \sin(s\theta + \alpha).$$

$s=0$

$$r = |r_{max}| \sin(\alpha) = r_0.$$

After 1 round trip:

$$r_1 = A r_0 + B r_0' = |r_{max}| \sin(\theta + \alpha).$$

$$\begin{aligned} A r_0 + B r_0' &= |r_{max}| \sin(\theta+\alpha). \\ &= |r_{max}| (\sin\theta\cos\alpha + \cos\theta \sin\alpha). \\ &= |r_{max}| \sin\theta \cos\alpha + |r_{max}| \frac{A+D}{2} \sin\alpha. \\ &= |r_{max}| \sin\theta \cos\alpha + r_0 \frac{(A+D)}{2} \end{aligned}$$

$$|r_{max}| \cos\alpha = \frac{1}{\sin\theta} \left[r_0 \left(\frac{A-D}{2}\right) + B r_0' \right].$$

We have $|r_{max}| \sin\alpha = r_0$.

$$\tan\alpha = r_0 \sin\theta \left[r_0 \left(\frac{A-D}{2}\right) + B r_0' \right]^{-1}.$$

but $\sin\theta = \left(1 - \left(\frac{A+D}{2}\right)^2\right)^{1/2}.$

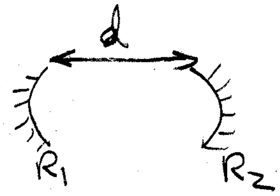
$$\tan\alpha = \left[\frac{r_0 \left(1 - \left(\frac{A+D}{2}\right)^2\right)^{1/2}}{r_0 \frac{(A-D)}{2} + B r_0'} \right]$$

Once α is determined

$$|r_{max}| = \frac{r_0}{\sin\alpha}.$$

① $\tan\alpha = \left[\dfrac{r_0\left(1-\left(\frac{A+D}{2}\right)^2\right)^{1/2}}{r_0\frac{(A-D)}{2} + Br_0'}\right]$

Review. Stability Criteria



$0 \le g_1 g_2 \le 1$ $\qquad g_{1,2} = 1 - \dfrac{d}{R_{1,2}}$

Solution for stable system $r = r_{max}\sin(s\theta + \alpha)$
where s is the number of round trips.

Unstable Cavities.

Recall difference equation is of form

$$r_{s+2} - 2\left(\frac{A+D}{2}\right) r_{s+1} + r_s = 0.$$

For unstable cavities, solution is of the form
$r_s = r_0 (F)^s$
both solutions are real.

$$F_{1,2} = \frac{A+D}{2} \pm \left[\left(\frac{A+D}{2}\right)^2 - 1\right]^{1/2}$$

General solution $\quad r_s = r_a (F_1)^s + r_b (F_2)^s$

Unstable cavities $\quad \underbrace{\dfrac{A+D}{2}}_{} \ge 1 \quad$ or $\quad -1 \ge \dfrac{A+D}{2}$

remember $\cos\theta$ for stable cavities.

$\Rightarrow$ F_1 or F_2 has a magnitude >1 and thus after a few round trips the solution is approximately
$r_s \sim r_0 (F_>)^s$
^ largest solution.

Evaluation of r_a & r_b.

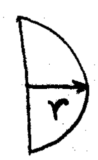
$$S=0 \qquad r_s = r_a + r_b = r_0$$
$$S=1 \qquad r_1 = r_a F_1 + r_b F_2 = A r_0 + B r_0'$$

Solving for r_a and r_b yields:

$$r_b = \frac{1}{F_1 - F_2}\left(r_0(F_1 - A) - B r_0'\right)$$

$$r_a = \frac{1}{F_2 - F_1}\left(r_0(F_2 - A) - B r_0'\right).$$

So far we have discussed ray tracing in homogeneous material systems. What happens in inhomogeneous, or lenslike media.

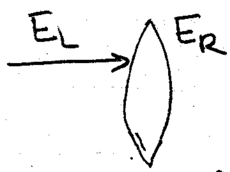


thickness causes changes to our simple minded picture.

plano-convex

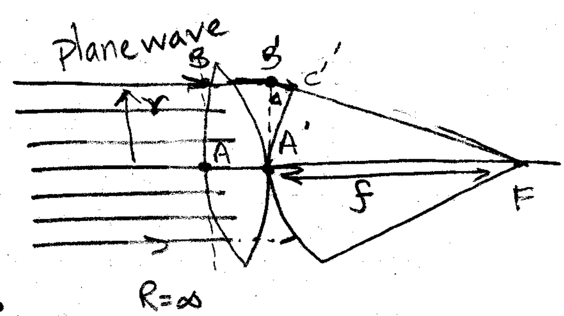
For example.

lens - constant index - quadratic physical path.



$$E_R(x,y) = E_L(x,y)\, e^{\frac{ik(x^2+y^2)}{2f}}$$

How do we get this?



If the lens is not there B' & A' would be in phase, however A' and C' are in phase

recall $v = \frac{c}{n}$.

$E \propto e^{-ikz}$ for right going wave

$k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0}$ ← index causes phase difference

$$\Delta = \overline{FB'} - \overline{FC'}$$

$$\overline{FB'} = \sqrt{r^2 + f^2} \qquad \overline{FC'} = f$$

$$\Delta = \overline{FB'} - \overline{FC'} = \sqrt{r^2+f^2} - f = f\sqrt{1+\frac{r^2}{f^2}} - f. \qquad \sqrt{1+\frac{r^2}{f^2}} \simeq 1 + \frac{r^2}{2f^2}$$

if $f \gg r$ (paraxial approximation).

$$\Delta \simeq f\left(1+\frac{r^2}{2f^2}\right) - f = \frac{r^2}{2f} \doteq \frac{x^2+y^2}{2f}$$

phase difference $= \#\lambda \times 2\pi = \frac{\text{path difference}}{\lambda} \times 2\pi = \Delta \times \frac{2\pi}{\lambda}$

$$\Rightarrow \quad E_R(x,y) = E_L(x,y)\, e^{ik\Delta} = E_L(x,y)\, e^{ik\left(\frac{x^2+y^2}{2f}\right)}.$$

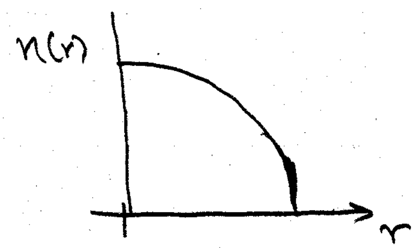
$v = \frac{c}{n}$ – wave inside glass is moving slower.

Another example.

$\boxed{n(r) \sim} \; r^2$ (distance from center).

eg. $n(x,y) = n_0\left[1 - \frac{k_2}{2k}(x^2+y^2)\right]$ $\quad k_2$ = constant of material

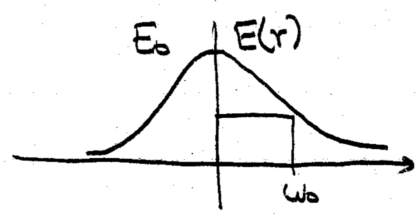
1. graded index fiber.



2. propagation of intense laser beam through a medium with $n = n(I)$.

$n = n_0 + n_2 I$

Gaussian spatial profile



$$I(r) = I_0\, e^{-2\left(\frac{r}{w_0}\right)^2}$$

w_0 = Half width at $\frac{E_0}{e}$.

paraxial approx:

$$I(r) = I_0 \left[1 - 2\left(\frac{r^2}{w_0^2}\right)\right] \qquad r \ll w_0.$$

$$\Rightarrow n = \underbrace{n_0 + n_2 I_0}_{n_0'} - 2 n_2 \frac{r^2}{w_0^2}$$ where n_2 is a constant of the material.

$$= n_0' - 2n_2\left(\frac{x^2+y^2}{w_0^2}\right) = n_0'\left[1 - \frac{2n_2}{n_0' w_0^2}(x^2+y^2)\right].$$

→ Gaussian beam propagating in weakly absorbing medium (change index due to heating).

$$\frac{dn}{dT} \neq 0$$

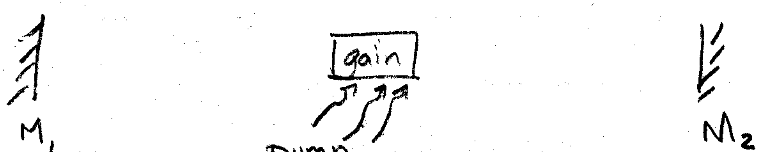
$\frac{dn}{dT} > 0$. (n increases in center of beam relative to wings ⇒ converging lens "self focusing".)

$\frac{dn}{dT} < 0$ ⇒ diverging lens "self-defocusing).

(3) Optical pumping of solid state lasers:

3 elements to laser

① gain medium
② pumping mechanism.
③ cavity. (resonator).



feed back.

- population inversion.

gain medium: gas, organic dyes, some solid state material

pumping: electrical discharge, chemical reaction, another laser, flash lamps.

Nd:YAG ← flashlamps

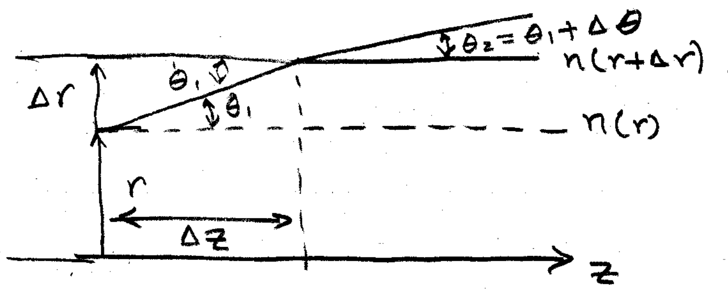
Ti:Sapphire ← laser.

Energy in gain medium not converted to laser $\Rightarrow$ heat
$\Rightarrow$ Temperature gradient
$T(0) > T(r_0)$ · temp at center greater than at outside.

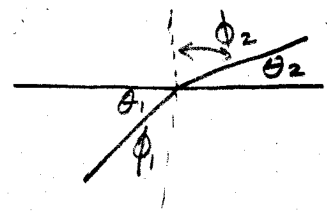
$\frac{dn}{dT} \neq 0 \Rightarrow$ lens effect..

General Differential equation describes propagation through inhomogeneous medium.

Paraxial approximation:



take $\Delta r \to 0$.
Snells' law of refraction.



$n_1 \sin\phi_1 = n_2 \sin\phi_2$

$$n_1 \sin\left(\frac{\pi}{2} - \theta_1\right) = n_2 \sin\left(\frac{\pi}{2} - \theta_2\right).$$

$$n_1 \cos(\theta_1) = n_2 \cos(\theta_2).$$

so: $$n(r)\cos(\theta_1) = n(r+\Delta r)\cos(\theta_1 + \Delta\theta).$$

expand $n(r)$ in Taylor series.

$$n(r+\Delta r) \cong n(r) + \frac{\partial n}{\partial r}\Delta r + \cdots\cdots$$

$$\cos(\theta_1 + \Delta\theta) = \cos\theta_1 \cos\Delta\theta - \sin\theta_1 \sin\Delta\theta.$$

$$n(r)\cos\theta = \left[n(r) + \frac{\partial n}{\partial r}\Delta r\right]\left[\cos\theta_1 \cos\Delta\theta - \sin\theta_1 \sin\Delta\theta\right].$$

for small $\Delta\theta$, $\cos\Delta\theta \simeq 1$, $\sin\Delta\theta \simeq \Delta\theta$.

$$n(r)\cancel{\cos\theta_1} = n(r)\cancel{\cos\theta_1} - n(r)\sin(\theta)(\Delta\theta) + \frac{\partial n}{\partial r}\Delta r\,(\cos\theta_1 - \cancel{\overset{\text{small}\to 0}{\Delta\theta}}\sin\theta_1).$$

$$\Rightarrow \quad n(r)\sin\theta_1\,\Delta\theta = \frac{\partial n}{\partial r}\Delta r\cos\theta_1.$$

$$\text{or} \quad \frac{\partial n}{\partial r} = n(r)\tan\theta_1\left(\frac{\Delta\theta}{\Delta r}\right).$$

but paraxial approx: $\tan\theta_1 \simeq \dfrac{\Delta r}{\Delta z}$

$$\frac{\partial n}{\partial r} = n(r)\frac{\Delta r}{\Delta z}\frac{\Delta\theta}{\Delta r} = n(r)\frac{\Delta\theta}{\Delta z}$$

$$\theta_1 \simeq \frac{\Delta r}{\Delta z} \Rightarrow \Delta\theta = \Delta\left(\frac{\Delta r}{\Delta z}\right) \to \frac{\Delta\theta}{\Delta z} = \frac{\Delta}{\Delta z}\left(\frac{\Delta r}{\Delta z}\right)$$

$$\text{as } \Delta\to 0 \Rightarrow \frac{\Delta\theta}{\Delta z} \to \frac{\partial^2 r}{\partial z^2}$$

$$\therefore \quad \frac{\partial n}{\partial r} = n(r)\frac{\partial^2 r}{\partial z^2} \qquad \boxed{\frac{\partial^2 r}{\partial z^2} = \frac{1}{n(r)}\frac{\partial n(r)}{\partial r}}$$

– solve this equation to describe ray motion in inhomogeneous medium.