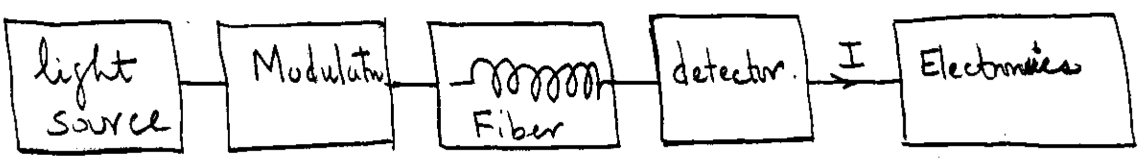
Some current uses:

① fiber optic communications
- low loss fiber
- high bandwidth.
- $\lambda \approx 1.5\,\mu m$ or $\lambda = 1.3\,\mu m$.
  $\nu = 200$ THz.
- soliton propagation (Nonlinear optics).

<u>What devices are necessary?</u>

① switches (routers).
② modulators
③ detectors
④ light source.

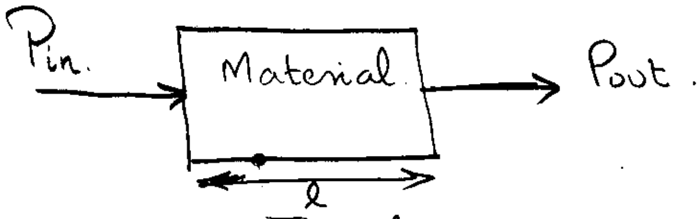


Laser
or LED Light emitting diode.

If transmitting logical signals (1's and 0's) we need only have a switch for the modulator.

Modulators: Black Box approach

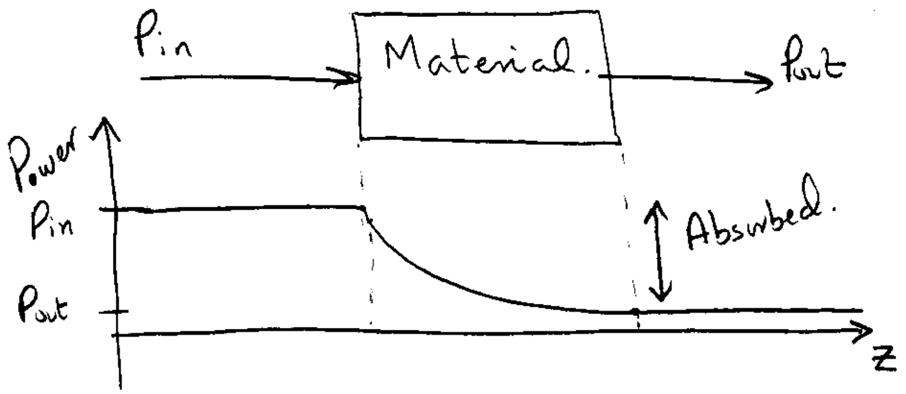
P_{in} - incident power.
P_{out} - exit power.



$$P_{out} = e^{-\alpha l} P_{in}$$

Electroabsorption Modulators. eg.

α : absorption coefficient
l : length of the medium.

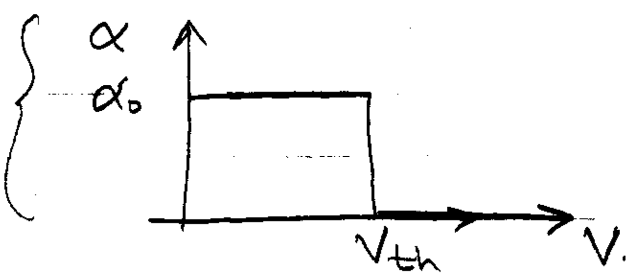


Power absorbed by medium.

$P_{out} = e^{-\alpha l} P_{in}$ Absorbed $= P_{in} - P_{out}$
$= (1 - e^{-\alpha l}) P_{in}$.

Suppose we have a material that α looks like this.

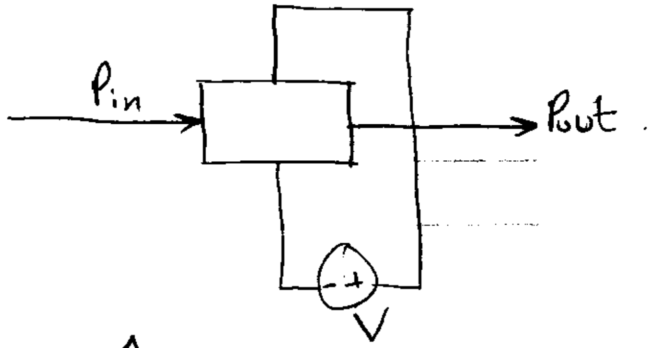
ideal.



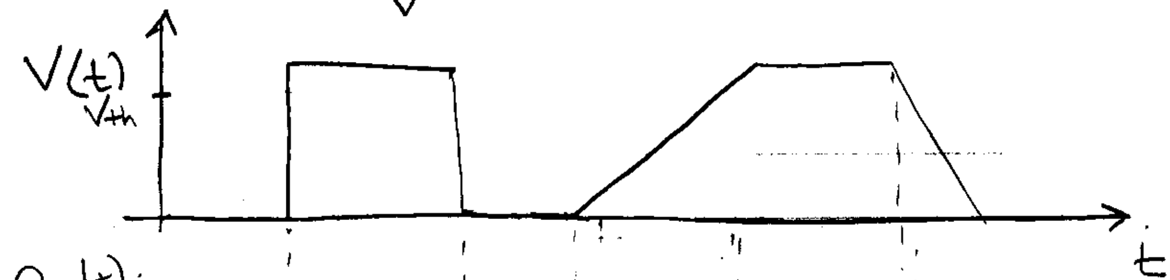
V - applied voltage.
Vth - threshold voltage.

That is, if I apply a voltage across the material, the absorption coefficient $\alpha = \alpha_0$ for $V < V_{th}$.

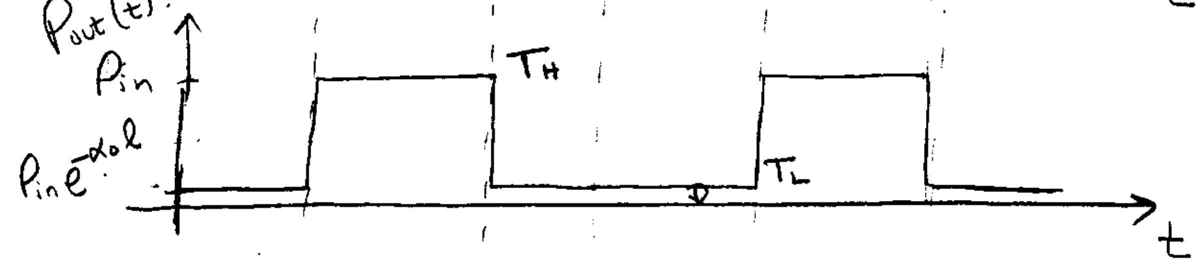
Then



Suppose



Then



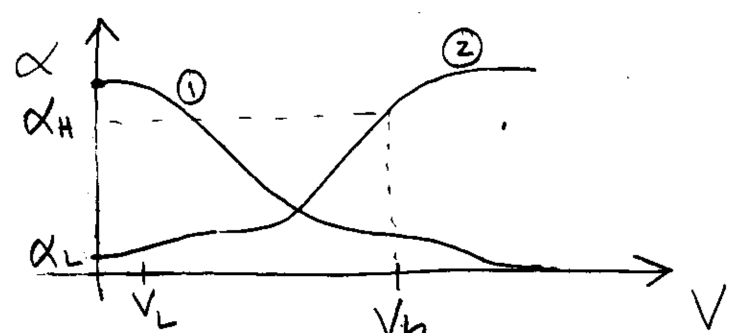
We have modulated the output by applying the voltage.

Go back to basics: we call the high state transmission

$$T_H = P_{out}^{high} = P_{in} \text{ (in this case).}$$
$$T_L = P_{out}^{Low} = P_{in}\, e^{-\alpha_0 l}$$

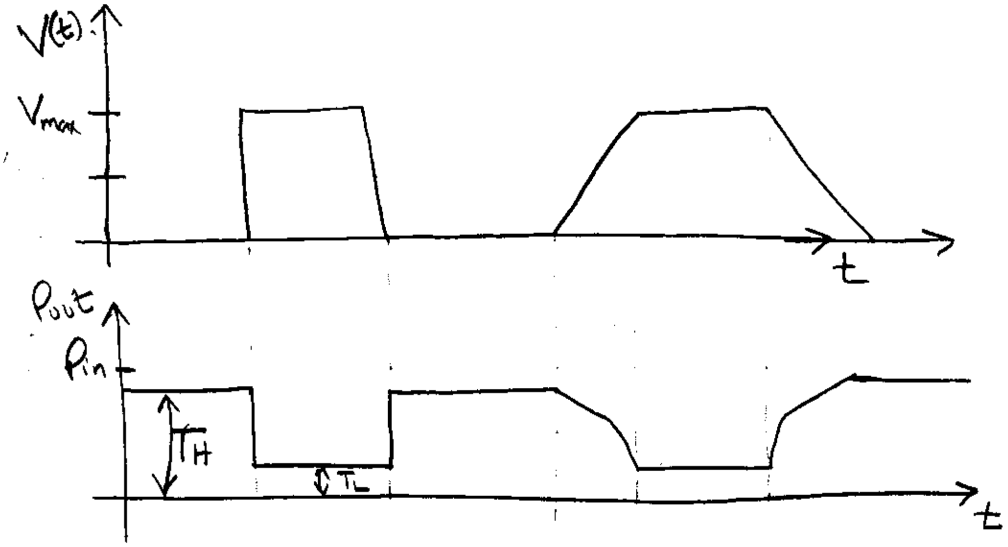
$$CR - \text{contrast ratio} = \frac{T_H}{T_L} = e^{\alpha_0 l}.$$

Reality α vs V



that is $\alpha(V) = f(V)$. (nonlinear)

For ②



$$CR: \quad \frac{T_H}{T_L} = \frac{P_{in} e^{-\alpha_l l}}{P_{in} e^{-\alpha_h l}} = e^{\underbrace{(\alpha_h - \alpha_l)}_{\Delta\alpha} l}$$

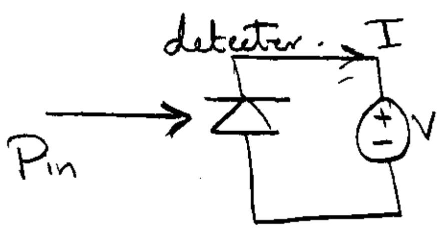
$$\Delta\alpha = \alpha_h - \alpha_l \quad \Rightarrow \quad CR = \underline{\underline{e^{\Delta\alpha l}}}$$

Large contrast ratios ⇒ Large $\Delta\alpha$.

For logic circuits $T_H \sim 1$. i.e. $(0.8\,T_H \to 1.2\,T_H)$
$T_L \sim 0$ $(< 1.5\,T_L)$.

Detector.

Ideal



$I = R P_{in}$. $\quad R$ - responsivity. (A/W).

many issues - see Dr. Andersons course.
(we will discuss this a little bit later in the course.)

Equations that govern light propagation.

Maxwells Equations.

Faradays law: $$\underbrace{\vec{\nabla} \times}_{\text{curl}} \vec{e} = -\frac{\partial \vec{b}}{\partial t} \quad (1)$$

Ampere's circuital law: $$\vec{\nabla} \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t} \quad (2)$$

Gauss law $$\underbrace{\vec{\nabla} \cdot}_{\text{divergence.}} \vec{d} = \rho \quad (3)$$

No isolated magnetic charge: $$\vec{\nabla} \cdot \vec{b} = 0 \quad (4)$$

$\vec{e}$: electric field intensity vector
$\vec{h}$: magnetic " " "
$\vec{b}$: magnetic flux density
$\vec{d}$: electric flux density (electric displacement)
: volume density of free charges.
$\vec{j}$ density of free currents

Equation of continuity $$\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t}.$$

Lorentzs force equation $$\vec{F} = q(\vec{e} + \vec{u} \times \vec{b}) \quad (N).$$
$\vec{u}$ is the velocity vector.

and constitutive relations
$$\vec{d} = \epsilon_0 \vec{e} + \vec{p} \qquad \vec{b} = \mu_0(\vec{h} + \vec{m}).$$

$\vec{p}$: polarization vector.

All vectors here are functions of time & space.
$$t, \quad \vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z.$$

Limit our interest to time-varying fields: (sinusoidal, just like circuits.).

$$\vec{e}(\vec{r},t) = Re\left\{\underbrace{\vec{E}(\vec{r})}_{\text{complex \# representing space variatn.}}\ \underbrace{e^{j\omega t}}_{\text{sinusoidal.}}\right\}$$

$$h(\vec{r},t) = Re\left\{\vec{H}(r)\, e^{j\omega t}\right\}$$

↓ all vectors the same.

Phasor representation. $\vec{a}(\vec{r},t) = Re\left[\vec{A}(\vec{r})\, e^{j\omega t}\right]$.

where $\vec{A}(\vec{r}) = \underbrace{|A(\vec{r})|}_{\text{magnitude}}\ \underbrace{e^{j\phi}}_{\text{phase}}$.

sometimes we write $\vec{a}(\vec{r},t) = A\, e^{j\omega t}$ it is understood that A is complex and we take the real part.

$$\vec{a}(\vec{r},t) = Re\left[|A(\vec{r})|\, e^{j\omega t} e^{j\phi}\right].$$

$$\boxed{e^{jx} = \cos x + j \sin x.} \Rightarrow a(\vec{r},t) = |A(\vec{r})| \cos(\omega t + \phi).$$

$$\therefore \frac{d}{dt} Re\left[\vec{A}(\vec{r})\, e^{j\omega t}\right] = Re\left\{j\omega \vec{A}(\vec{r})\, e^{j\omega t}\right\}$$

$$= Re\left\{j\omega |A|\, e^{j(\omega t+\phi)}\right\} = Re\left\{|A|\left(j\omega \cos(\omega t+\phi) - \omega \sin(\omega t+\phi)\right)\right\}$$

$$= -|A|\omega \sin(\omega t + \phi).$$

Lets use $\frac{d}{dt}\vec{a}(\vec{r},t) = \left\{j\omega\, \vec{A}(\vec{r})\, e^{j\omega t}\right\}$ in Maxwells equatns.

(1) $\nabla \times \vec{e} = -\dfrac{\partial \vec{b}}{\partial t}$ $\quad \vec{\nabla} \times Re\left\{\vec{E}(\vec{r})\, e^{j\omega t}\right\} = -\dfrac{\partial}{\partial t}\left(Re\left\{B(\vec{r})\, e^{j\omega t}\right\}\right)$.

$$\nabla \times \vec{E}(\vec{r})\, e^{j\omega t} = -j\omega \vec{B}(\vec{r})\, e^{j\omega t}$$

$$\nabla \times \vec{E}(\vec{r}) = -j\omega \vec{B}(r). \qquad (A)$$

similarly $\vec{\nabla}\times\vec{H}(\vec{r}) = J(\vec{r}) + j\omega D(\vec{r})$ (B)

$$\vec{D}(\vec{r}) = \epsilon_0 \vec{E}(\vec{r}) + \vec{P}(\vec{r}).$$

$$\nabla\cdot D(\vec{r}) = \rho \quad \text{(C)}$$

$$\nabla\cdot B(\vec{r}) = 0 \quad \text{(D)}.$$

Still too complicated: simplify by assuming free space.

$\vec{m}, \vec{J}, \vec{P}$ and $\rho = 0$. (linear isotropic).

$$\Rightarrow \nabla\times\vec{E} = -j\omega\vec{B}. \quad \text{(A)} \qquad \vec{B} = \mu_0\vec{H}$$

$$\nabla\times\vec{H} = j\omega\vec{D} \qquad \vec{D} = \epsilon_0 E$$

$$\nabla\times\vec{H} = j\omega\epsilon_0\vec{E} \qquad \text{(B)}$$

Lets take $\nabla\times(\text{(A)}) \Rightarrow \nabla\times\nabla\times\vec{E} = -j\omega\nabla\times\vec{B} = -j\omega\mu_0\nabla\times\vec{H}$.

$$\nabla\times\nabla\times\vec{E} = -j\omega\mu_0\left(j\omega\epsilon_0\vec{E}\right) = \omega^2\mu_0\epsilon_0\vec{E}.$$

but

$$\nabla\times\nabla\times\vec{E} = \nabla(\underbrace{\nabla\cdot\vec{E}}_{=0}) - \nabla^2\vec{E}$$

$$\Rightarrow -\nabla^2 E = \omega^2\mu_0\epsilon_0\vec{E} \quad \text{or} \quad \nabla^2\vec{E} + \omega^2\mu_0\epsilon_0\vec{E} = 0.$$

but $c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ so $\nabla^2\vec{E} + \frac{\omega^2}{c^2}\vec{E} = 0$ (time-independent wave equation.)

We could have done this in time domain (original equation 1-4.)

We would have

$$\left.\begin{aligned} &\nabla^2\vec{e} - \frac{1}{c^2}\frac{\partial^2\vec{e}}{\partial t^2} = 0. \\ \text{for } \vec{h} \quad &\nabla^2\vec{h} - \frac{1}{c^2}\frac{\partial^2\vec{h}}{\partial t^2} = 0. \end{aligned}\right\} \quad \text{Next time solutions.}$$

Motivation for photonic devices. Why?

First - photon - fundamental quanta of light
smallest bundle of EM energy.

$$E_p = \frac{hc}{\lambda} = h\nu \qquad \nu = \frac{c}{\lambda}$$

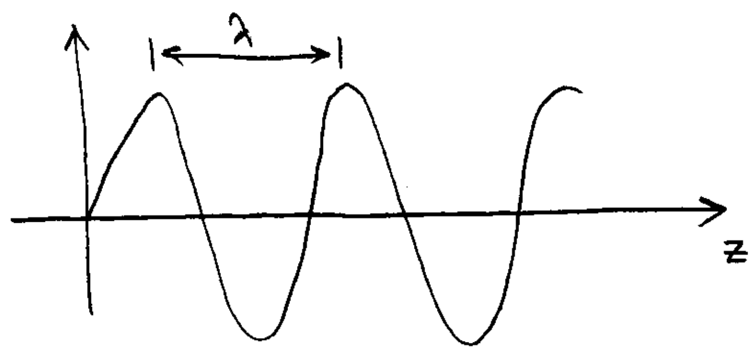
E_p : Energy of the photon

h : Planck's constant. 6.6262×10^{-34} J.s.

c : speed of light 3×10^{8} m/s

λ : wavelength of oscillation.

ν : frequency of oscillation.



Notice shorter wavelength $\Rightarrow$ more energy. & higher frequency.

What magnitude of frequencies are we considering?

Most photonic devices are fabricated w/ GaAs $\Rightarrow \lambda \sim 800$nm.

Gallium Arsenic

1 nm = 10^{-9} meters.

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{m.}}{0.8 \times 10^{-6}} = 3.75 \times 10^{14} \text{ Hz.}$$

or 375 THz.

From handout we know transmission of current TV, radar, air traffic control etc is in UHF & VHF

(300-3000 MHz) (30-300 MHz)

1 TV channel has a band width of $\sim 1 \times 10^6$ Hz 1MHz.

From basic communications theory: Maximum (theoretical) number of channels we can transmit is.

$$\sim \frac{f_c}{BW} - \frac{\text{carrier frequency}}{\text{Band-width}} = \frac{3000\text{MHz}}{1\text{MHz}} \sim 3000$$

However with optical beam @ 800nm we can have

$$\frac{3.75 \times 10^{14}}{1 \times 10^6} = 3.75 \times 10^8 \text{ channels.}$$

375 million channels.

Last Time. Maxwells Equations.

$$\vec{\nabla}\times\vec{e} = -\frac{\partial \mathfrak{b}}{\partial t} \qquad \vec{\nabla}\times \mathfrak{h} = \vec{j} + \frac{\partial \mathfrak{d}}{\partial t}$$

$$\nabla\cdot\mathfrak{d} = \rho \qquad \vec{\nabla}\cdot\mathfrak{b} = 0,$$

continuity: $\nabla\vec{j} = -\dfrac{\partial\rho}{\partial t}$

Force Eqn: $\vec{F} = q(\vec{e} + \vec{u}\times\mathfrak{b})$ (N).

$$\vec{d} = \epsilon\vec{e} + \vec{p} \qquad \mathfrak{b} = \mu(\vec{h} + \vec{m}).$$

Using these we derived the wave eqn for the electric field:

$$\nabla^2\vec{E} + \frac{\omega^2}{c^2}\vec{E} = 0 \qquad \text{(time independent wave eqn.)}.$$

$$\nabla^2\vec{H} + \frac{\omega^2}{c^2}\vec{H} = 0.$$

$$\nabla^2\mathfrak{h} - \frac{1}{c^2}\frac{\partial^2\mathfrak{h}}{\partial t^2} = 0 \qquad \text{(time dependent)}.$$

$$\nabla^2\vec{e} - \frac{1}{c^2}\frac{\partial^2\vec{e}}{\partial t^2} = 0.$$

What do these mean?

Consider $\nabla^2\vec{e} - \dfrac{1}{c^2}\dfrac{\partial^2\vec{e}}{\partial t^2} = 0.$

What are the expected solutions?
In one dimension, this equation is

$$\frac{\partial^2 e_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0.$$

We say any $f(t - \frac{x}{c})$ is a solution.

Proof.
$$\frac{\partial^2 f(t-\frac{x}{c})}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f(t-\frac{x}{c})}{\partial t^2} = 0.$$

Recall $\frac{d}{dx} f(u(x)) = \frac{d f(u(x))}{d u(x)} \cdot \frac{d u(x)}{dx}.$

$$\therefore \frac{\partial}{\partial x} f(t-\frac{x}{c}) = \frac{d f(u)}{du}\left(-\frac{1}{c}\right)$$

$$\frac{\partial^2}{\partial x^2} f(t-\frac{x}{c}) = \frac{d}{\partial x}\left(\quad \right) = \frac{1}{c^2}\frac{d^2 f(u(x))}{du^2}.$$

Similarly $\frac{1}{c^2}\frac{\partial^2 f(t-\frac{x}{c})}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 f(u(x))}{\partial u^2}$

$$\Rightarrow \quad \frac{1}{c^2}\frac{\partial^2 f(u(x))}{\partial u^2} - \frac{1}{c^2}\frac{\partial^2 f(u(x))}{\partial u^2} = 0$$

One possible solut: $e_x = A_0 \cos(t - \frac{x}{c})$.

$$\frac{d^2 e_x}{dx^2} = -\frac{A_0}{c^2}\cos\left(t-\frac{x}{c}\right). \quad \frac{d^2}{dt^2} e_x = -\frac{A_0}{c^2}\cos\left(t-\frac{x}{c}\right).$$

$\Rightarrow$ this is a solution!

Suppose we limit ourselves to time harmonic fields, i.e, phasors.

$$\begin{Bmatrix}\vec{e}\\ \vec{h}\end{Bmatrix} = \mathrm{Re}\left(\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix}\exp(j\omega t)\right).$$

This means wave equat:

$$\nabla^2 E + \frac{\omega^2}{c^2} E = 0.$$

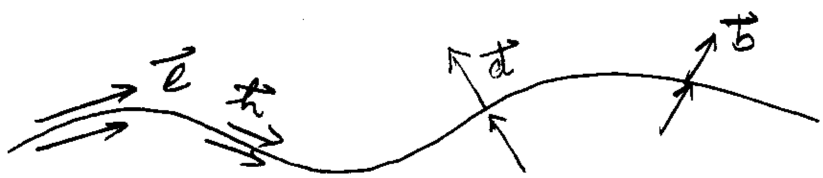
Without any proof:

Intensity & Power: $\vec{S} = \vec{e} \times \vec{h}$: Poynting vector

direction of power flow is along the direction of the Poynting vector. (orthogonal to both $\vec{e}$ & $\vec{h}$).

I : power flow across a unit area normal to the vector $\vec{S}$. time-averaged value of $\vec{S}$ - averaged over times > optical cycle but < times of interest.

Boundary conditions. - interfaces.



tangential components of $\vec{e}$ & $\vec{h}$ and normal components of $\vec{d}$ & $\vec{b}$ are continuous at boundaries of two different media without free charge or currents.

Dielectric Media.

$$e(\vec{r},t) \longrightarrow \boxed{\text{Medium}} \longrightarrow \vec{p}(\vec{r},t)$$

↑ response to $\vec{e}(r,t)$.

Useful definitions.

① Linear : $\vec{p}(\vec{r},t) = \underbrace{\epsilon_0 \chi}_{\text{constants}} \vec{e}$ — superposition applies.

② nondispersive : $\vec{p}(\vec{r},t)$ depends on $\vec{e}(\vec{r},t)$ and not on $\vec{e}(\vec{r},t_0)$ (some other time) - idealization all systems have some response time (but very short ~ 100fs).

③ homogeneous - $\vec{p}(\vec{r},t) = \underbrace{\epsilon_0 \chi}_{\text{does not depend on } \vec{r}.\ (\text{position})} \vec{e}(\vec{r},t)$

④ isotropic - relationship between $\vec{p}$ & $\vec{e}$ are independent of direction

i.e., $\chi \neq \vec{\chi}$ vector.

⑤ spatially nondispersive ⇒ $\vec{p}$ and $\vec{e}$ relationship is local i.e., $\vec{e}$ at point $\vec{r}$ does not generate a $\vec{p}$ at point $\vec{r}_0$.

∴ Linear, Nondispersive, Homogeneous and Isotropic Media.

$$\vec{p} = \epsilon_0 \chi \vec{e}$$

χ - scalar constant - electric susceptibility.

$$\vec{e} \longrightarrow \boxed{\chi} \longrightarrow \vec{p}$$

Notice: $\vec{d} = \epsilon_0 \vec{e} + \vec{p} = \epsilon_0 \vec{e} + \epsilon_0 \chi \vec{e} = \underbrace{\epsilon_0 (1+\chi)}_{\epsilon} \vec{e}$.

ϵ : electric permittivity of the medium.

$\frac{\epsilon}{\epsilon_0} = \epsilon_r$: dielectric constant.

In this case: $\nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \qquad u : \vec{e}, \vec{h}$.

where $c = \frac{c_0}{n}$ where $n = \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} = (1+\chi)^{1/2}$. $c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$.

n - refractive index.

n - refractive index is the square root of the dielectric constant of the material.

Next media $\Longrightarrow$ of most interest in this class:

Nonlinear, dispersive, inhomogenous or Anisotropic - Media

Inhomogenous dielectric medium. - eg. graded-index medium. (linear, nondispersive, and isotropic).

We still say.

$$\vec{p} = \epsilon_0 \chi \vec{e} \quad \text{and} \quad \vec{d} = \epsilon \vec{e}$$

but $\chi = \chi(\vec{r})$ and $\epsilon = \epsilon(\vec{r})$. $\Rightarrow n = \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} = \sqrt{\epsilon_r} = n(\vec{r})$.

Wave equation?

$$\nabla\cdot\vec{d} = 0 \quad (1) \qquad \vec{\nabla}\times\vec{h} = \frac{\partial \vec{d}}{\partial t} \quad (3) \qquad \vec{b} = \mu_0 \vec{h}$$

$$\vec{\nabla}\times\vec{e} = -\frac{\partial \vec{b}}{\partial t} \quad (2) \qquad \vec{d} = \epsilon(\vec{r})\vec{e}.$$

$$\Rightarrow \nabla\times\nabla\times\vec{e} = -\mu_0 \frac{\partial^2}{\partial t^2}\vec{d} = \nabla(\nabla\cdot\vec{e}) - \nabla^2\vec{e} = -\nabla^2\vec{e} + \nabla(\nabla\cdot\vec{e}).$$

$$\therefore \nabla^2\vec{e} - \mu_0\frac{\partial^2 \vec{d}}{\partial t^2} - \nabla(\nabla\cdot\vec{e}) = 0 \quad \left(\vec{d} = \epsilon_0\vec{e} + \vec{p} = \epsilon(\vec{r})\vec{e} + \epsilon(\vec{r})\chi(\vec{r})\vec{e} = \epsilon(\vec{r})\left(1 + \chi(\vec{r})\right)\vec{e}.\right)$$

$$\nabla\cdot\vec{d} = \nabla\cdot\epsilon\vec{e} = \epsilon\nabla\cdot\vec{e} + \vec{e}\cdot\nabla\epsilon = 0$$

$$\text{or} \quad \nabla\cdot\vec{e} = -\frac{1}{\epsilon}(\nabla\epsilon\cdot\vec{e})$$

$$\text{or} \quad \nabla^2\vec{e} - \frac{1}{c^2(\vec{r})}\frac{\partial^2\vec{e}}{\partial t^2} + \nabla\left(\frac{1}{\epsilon}\nabla\epsilon\cdot\vec{e}\right) = 0. \qquad c(\vec{r}) = \frac{1}{\sqrt{\mu_0\,\epsilon(\vec{r})}}$$

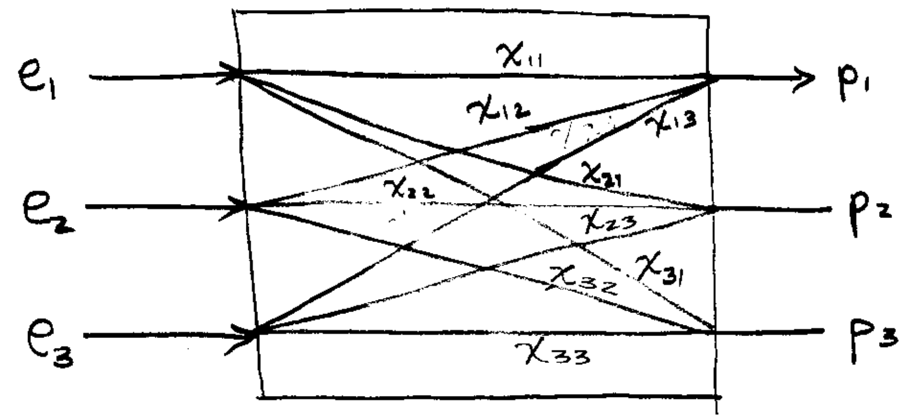
Complicated $\Rightarrow$ approximate: if $\epsilon(\vec{r})$ varies in space much slower than $\vec{e}(\vec{r},t)$, i.e, if $\epsilon(\vec{r})$ does not vary significantly within a wavelength distance,

$$\nabla\left(\frac{1}{\epsilon(r)}\nabla\epsilon(r)\cdot\vec{e}\right) \ll \nabla^2\vec{e}.$$

$$\Rightarrow \quad \nabla^2\vec{e} - \frac{1}{c^2(\vec{r})}\frac{\partial^2\vec{e}}{\partial t^2} = 0$$ is the correct wave eqn.

Anisotropic media. (linear, homogeneous, nondispersive).

$\vec{P}$ depends on the direction of vector $\vec{e}$.



$$p_i = \sum_j \epsilon_0 \chi_{ij} e_j$$ $i, j = 1, 2, 3$ denote x, y, z components

χ_{ij} : 3X3 : susceptibility tensor.

$$d_i = \sum_j \epsilon_{ij} e_j$$ $\{\epsilon_{ij}\}$ electric permittivity tensor.

$\Rightarrow$ discussed later. (in detail).

Dispersive media. - system has memory.

$$e(t) \longrightarrow \boxed{\chi(t)} \longrightarrow p(t).$$

$$p(t) = \epsilon_0 \int_{-\infty}^{\infty} \chi(t-t')\, e(t')\, dt'$$ convolution.

<u>Nonlinear Media</u> – of course we should be interested in this.

For example $\vec{p} = a\vec{e} + a_2 e^2 + a_3 e^3$ (non dispersive but non linear).
wave equation not applicable.

However there is a nonlinear partial differential equation that governs it!

Where do we start? Maxwells equations, of course!

$$\vec{p} = \psi(\vec{e}) \quad \text{(some nonlinear function)}.$$

still: $\nabla \times \nabla \times \vec{e} = -\mu \dfrac{\partial^2 d}{\partial t^2}$ same to here.

$\vec{d} = \epsilon \vec{e} + \vec{p}$ and $\nabla \times \nabla \times \vec{e} = \nabla(\nabla \cdot \vec{e}) - \nabla^2 e$.

$$\nabla(\nabla \cdot \vec{e}) - \nabla^2 e = -\mu\epsilon \frac{\partial^2 \vec{e}}{\partial t^2} - \mu \frac{\partial^2 \vec{p}}{\partial t^2}.$$

Suppose. homogeneous & isotropic $\Rightarrow \vec{d} = \epsilon \vec{e}$ & $\nabla \cdot d = 0$.
$\Rightarrow \nabla \cdot \vec{e} = 0$. $(\epsilon \neq \epsilon(\vec{r}))$.

Substitute $\nabla \cdot \vec{e} = 0$ & $\mu\epsilon = \dfrac{1}{c^2}$.

$$\rightarrow \nabla^2 \vec{e} - \frac{1}{c^2}\frac{\partial^2 \vec{e}}{\partial t^2} = \mu \frac{\partial^2 \vec{p}}{\partial t^2} \quad \Big\} \text{ homogeneous \& isotropic}.$$

if in addition, media is nondispersive i.e, $p = \psi(\vec{e}) = a_1 e + a_2 e^2 + a_3 e^3$
where a_1, a_2, a_3 are constants.

$$\Rightarrow \nabla^2 \vec{e} - \frac{1}{c^2}\frac{\partial^2 e}{\partial t^2} = \mu \frac{\partial^2 \psi(e)}{\partial t^2}.$$

<u>Important</u>: Superposition no longer applies!

Monochromatic Electromagnetic waves.

– sinusoidal solutions.

$$e(\vec{r},t) = Re[\vec{E}(\vec{r})\exp(j\omega t)].$$

$\Rightarrow$ Maxwells equations

$$\vec{\nabla}\times\vec{H} = j\omega\vec{D} \qquad \vec{D} = \epsilon_0\vec{E} + \vec{P}$$
$$\nabla\times\vec{E} = -j\omega\vec{B} \qquad B = \mu_0\vec{H}$$
$$\nabla\cdot\vec{D} = 0$$
$$\nabla\cdot\vec{B} = 0$$

Optical Intensity and Power

$$S = \vec{e}\times\vec{h}$$

$$S = Re\{\vec{E}e^{j\omega t}\}\times Re\{\vec{H}e^{j\omega t}\} = \frac{1}{2}\left(\vec{E}e^{j\omega t} + \vec{E}^*e^{-j\omega t}\right)\times\frac{1}{2}\left(\vec{H}e^{j\omega t} + \vec{H}^*e^{-j\omega t}\right)$$

$$\underbrace{\left(Re(\vec{A}) = \frac{1}{2}(\vec{A}+\vec{A}^*)\right)}_{\text{you prove.}}$$

$$S = \frac{1}{4}\left(\vec{E}\times\vec{H}^* + E^*\times\vec{H} + \overbrace{\vec{E}\times\vec{H}e^{j2\omega t} + E^*\times H^* e^{-j2\omega t}}^{\text{high frequency.}}\right).$$

$$\langle S\rangle = \frac{1}{T}\int_0^T S\,dt \Rightarrow \text{high freq terms average out.}$$

$$\therefore \langle S\rangle = \frac{1}{4}\left(\vec{E}\times\vec{H}^* + E^*\times\vec{H}\right) = \frac{1}{2}\left(\vec{S}_c + \vec{S}_c^*\right) = Re\{\vec{S}_c\}.$$

$$\vec{S}_c = \frac{1}{2}\vec{E}\times\vec{H}^* \quad \text{— complex poynting vector.}$$

$$I = |Re\{\vec{S}_c\}| = |\langle S\rangle|. \qquad \left(\text{we will evaluate later}\right).$$

Linear, nondispersive, homogenous and isotropic media.

$$\Rightarrow \vec{D} = \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P} \qquad \vec{B} = \mu_0 H$$

$\vec{E}$ & $\vec{H}$ must satisfy

$$\Rightarrow \underbrace{\nabla^2\vec{U} + k^2\vec{U} = 0}_{\text{Helmholtz equation}} \qquad k = \omega(\epsilon\mu_0)^{1/2} = nk_0 \qquad k_0 = \frac{\omega}{c_0}$$

Inhomogenuous Media $\epsilon = \epsilon(\vec{r})$.

for locally homogenous media.

Helmholtz Eqn is approximately correct with $\epsilon = \epsilon(\vec{r})$.

Dispersive media.

$$P(t) = \underbrace{\epsilon_0 \int_{-\infty}^{\infty} \chi(t-t')\, e(t')\, dt'}_{\text{convolution in time}}$$

$$\Rightarrow \vec{P} = \epsilon_0 \chi(\nu) \vec{E} \qquad \text{(Fourier transform pair).}$$

where $\chi(\nu) = \int_{-\infty}^{\infty} \chi(t)\, e^{-j2\pi\nu t}\, dt$ is the Fourier transform of $\chi(t)$.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\Rightarrow \vec{D} = \epsilon_0 (1 + \chi(\nu)) \vec{E} = \epsilon(\nu) \vec{E}$$

↑ frequency dependent!

$$\epsilon(\nu) = \epsilon_0 (1 + \chi(\nu))$$

Helmholtz equation holds! $\epsilon \to \epsilon(\nu)$.

$$k = \omega (\epsilon(\nu) \mu_0)^{1/2} = n(\nu) k_0.$$

$$n(\nu) = \left(\frac{\epsilon(\nu)}{\epsilon_0} \right)^{1/2} \text{ - frequency dependent.}$$

If $\chi(\nu)$, $\epsilon(\nu)$ and $n(\nu)$ are ~ constant in the frequency range of interest — approximate material as nondispersive.

Elementary EM waves.

- linear, homogenous and isotropic media.
- solutions to the wave equation.

① Transverse Electromagnetic Plane Wave (TEM wave).

$$\vec{E}(\vec{r}) = \vec{E}_0 \exp(-j\vec{k}\cdot\vec{r}).$$
$$\vec{H}(\vec{r}) = \vec{H}_0 \exp(-j\vec{k}\cdot\vec{r})$$

$\vec{k}$ - wave vector, $\vec{E}_0$ and $\vec{H}_0$ are constant vectors.
and $|\vec{k}| = k = n k_0$. k_0 - free space wave vector $= \frac{\omega}{c_0} = \frac{2\pi\nu}{c_0}$
$= \frac{2\pi}{\lambda_0}$.

$\vec{k}$ — normal to the plane wavefronts.

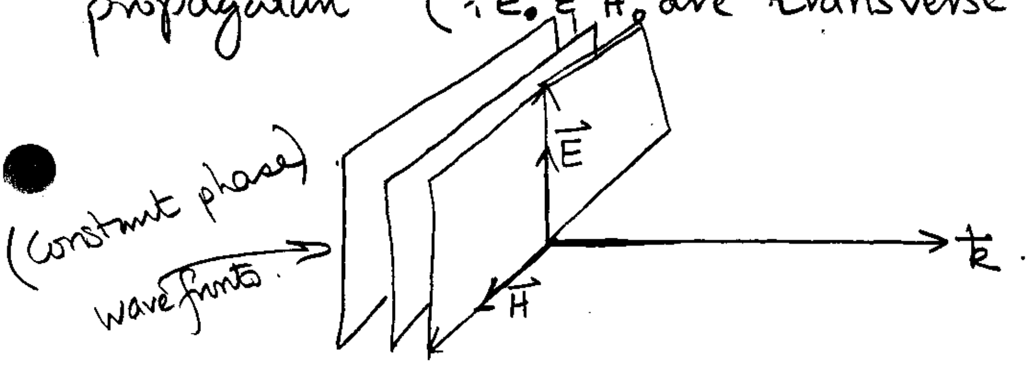
with this chose for the vectors we can see that

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} \qquad \nabla \times \vec{H} = \vec{k} \times \vec{H}.$$
$$\nabla \times \vec{E} = -j\omega\mu_0\vec{H} \qquad \nabla \times \vec{E} = \vec{k} \times \vec{E}$$

(Prove to yourself that $\nabla \times \vec{A}_0 e^{-j\vec{k}\cdot\vec{r}} = -j\vec{k} \times \vec{A}_0 e^{-j\vec{k}\cdot\vec{r}}$) use 22-39 ∴ $\vec{A}$: constant vector.

$$\therefore \left.\begin{aligned} \vec{k} \times \vec{H}_0 &= -\omega\epsilon\vec{E}_0 \\ \vec{k} \times \vec{E}_0 &= \omega\mu_0\vec{H}_0. \end{aligned}\right\} \Rightarrow \vec{k} \perp \vec{E}_0 \perp \vec{H}_0 \,!$$

$\vec{E}$ & $\vec{H}$ are normal to each other and to the direction of propagation (∴ $\vec{E}_0$ & $\vec{H}_0$ are transverse to $\vec{k}$).



What else do we know? $\vec{k} \times \vec{H}_0 = -\omega \epsilon \vec{E}_0$ ①
$\vec{k} \times \vec{E}_0 = \omega \mu_0 \vec{H}_0$ ②

- For vectors $\vec{k} \times \vec{H} = k \mathcal{H}_0 \sin\theta \, \hat{u}$ $\theta = 90°$
$= k \mathcal{H}_0 \hat{u}$ $k = |\vec{k}|$ $\mathcal{H}_0 = |\vec{H}|$.

$\Rightarrow$ $k\mathcal{H}_0 = \omega \epsilon \mathcal{E}_0$
or $\mathcal{H}_0 = \frac{\omega \epsilon}{k} \mathcal{E}_0$.

from ② $k \mathcal{E}_0 = \omega \mu_0 \mathcal{H}_0$ $\mathcal{H}_0 = \frac{k}{\omega \mu_0} \mathcal{E}_0$.

$\Rightarrow \frac{k}{\omega \mu_0} = \frac{\omega \epsilon}{k}$ or $k = \omega (\epsilon \mu_0)^{1/2}$.

Notice that $\frac{\mathcal{E}_0}{\mathcal{H}_0} = \frac{\omega \mu_0}{k} = \frac{\mu_0 \omega}{\omega (\epsilon \mu_0)^{1/2}} = \left(\frac{\mu_0}{\epsilon}\right)^{1/2}$

$\epsilon = \epsilon_0 (1 + \chi)$ $n = \sqrt{\epsilon_r} = \sqrt{1+\chi}$.

- $\therefore \frac{\mathcal{E}_0}{\mathcal{H}_0} = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \frac{1}{n} = \eta$. where $\eta = \frac{\eta_0}{n}$.

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \simeq 120\pi = 377 \, \Omega$. is the impedance of free space.

Itensity. $I = |\vec{S}| = \frac{1}{2} |\vec{E} \times \vec{H}^*| = \frac{1}{2} \mathcal{E}_0 \mathcal{H}_0^*$ or $\frac{1}{2} \frac{|\mathcal{E}_0|^2}{\eta}$

$$\boxed{I = \frac{|\mathcal{E}_0|^2}{2\eta}}$$ for TEM waves.

In free space $10 \, W/cm^2 \Rightarrow \mathcal{E}_0 = 87 \, V/cm$.

- For my laser $\frac{1W}{\pi \underbrace{(100 \times 10^{-4})^2}_{\text{typical spot size}}} \sim 3.2 \, kV/cm$. (if we assume a plane wave of CW light) (average field).

Two additional typical solutions.

① Spherical wave
② Gaussian beam.

Absorption and Dispersion.

We phenomenologically represent the complex susceptibility as

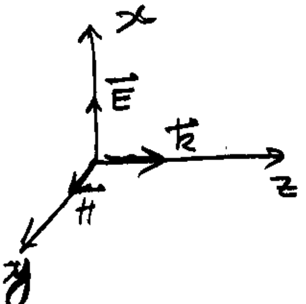
$$\chi = \chi' + j\chi'' \quad \text{for a dielectric material.}$$

and complex permittivity $\epsilon = \epsilon_0 (1+\chi)$.

Helmholtz equation still holds but $k = \omega(\epsilon\mu_0)^{1/2} = (1+\chi)^{1/2} k_0$
$= (1+\chi' + j\chi'')^{1/2} k_0$.

k - is now complex-valued

Plane wave traveling in z direction is described by.



$$\vec{E}(\vec{r}) = A \exp(-jkz)\,\hat{x}$$

$$= A \exp\Big(-j\underbrace{(1+\chi'+j\chi'')^{1/2} k_0}_{k} z\Big)\,\hat{x}.$$

We write $k = \beta - j\frac{\alpha}{2}$

$$\Rightarrow \vec{E}(\vec{r}) = A \exp(-j\beta z)\,\underbrace{\exp\left(-\frac{\alpha}{2}z\right)}_{\text{attenuation}}\,\hat{x}.$$

$$I = \frac{|E_0|^2}{2\eta} = \frac{|A|\,e^{-\alpha z}}{2\eta}.$$

α - absorption coefficient (attenuat coefficient)
(or extinction coefficient)

β – is the rate of phase change – propagat. constant.

Recall that previously we said n – refractive index

$$e^{-jkz} \qquad k = \omega(\epsilon\mu_0)^{1/2} = \frac{\omega}{c}n.$$
$$= \frac{2\pi n}{\lambda}. = nk_0$$

Now $e^{-jkz} \rightarrow e^{-j\beta z} \Rightarrow \beta = nk_0$. (definition).

wave travels with phase velocity $c = \frac{c_0}{n}$.

$$\therefore \quad nk_0 - j\frac{\alpha}{2} = (1 + \chi' + j\chi'')^{1/2} k_0.$$

$$n - \frac{j\alpha}{2k_0} = (1 + \chi' + j\chi'')^{1/2}.$$

For weakly absorbing Media: $\chi' << 1$ and $\chi'' << 1$.
and. $\sqrt{1+x} = 1 + \frac{1}{2}x$. for small x.

then $\sqrt{1+\chi'+j\chi''} \simeq 1 + \frac{1}{2}(\chi' + j\chi'') = n - \frac{j\alpha}{2k_0}$.

or $\quad n \simeq 1 + \frac{1}{2}\chi'$

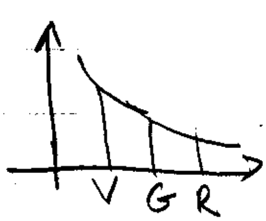
$$\alpha \simeq -k_0\chi''.$$

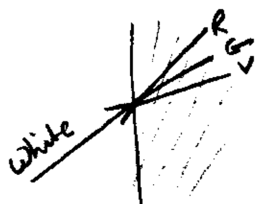
that is n is related to the real part of the susceptibility and α related to the imaginary part of the susceptibility, χ.

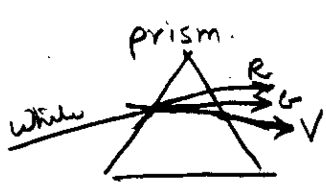
$$\chi' \leq 0 \Rightarrow \alpha > 0 \Rightarrow \text{absorption}.$$
$$\chi'' > 0 \Rightarrow \alpha < 0 \Rightarrow \text{gain}.$$

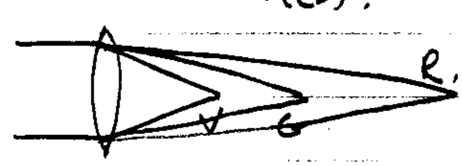
we will come back to this.

Dispersion. $\chi(\nu) \rightarrow n(\nu) = \sqrt{1 + \chi(\nu)}$. so $c(\nu) = \frac{c_0}{n(\nu)}$.









are the parametric equations of the ellipse,

$$\frac{\mathcal{E}_x^2}{a_x^2}+\frac{\mathcal{E}_y^2}{a_y^2}-2\cos\varphi\,\frac{\mathcal{E}_x\mathcal{E}_y}{a_x a_y}=\sin^2\varphi, \tag{6.1-5}$$

where $\varphi=\varphi_y-\varphi_x$ is the phase difference.

At a fixed value of z, the tip of the electric-field vector rotates periodically in the x–y plane, tracing out this ellipse. At a fixed time t, the locus of the tip of the electric-field vector follows a helical trajectory in space lying on the surface of an elliptical cylinder (see Fig. 6.1-1). The electric field rotates as the wave advances, repeating its motion periodically for each distance corresponding to a wavelength $\lambda = c/\nu$.

The state of polarization of the wave is determined by the shape of the ellipse (the direction of the major axis and the ellipticity, the ratio of the minor to the major axis of the ellipse). The shape of the ellipse therefore depends on two parameters—the ratio of the magnitudes a_y/a_x and the phase difference $\varphi=\varphi_y-\varphi_x$. The size of the ellipse, on the other hand, determines the intensity of the wave $I=(a_x^2+a_y^2)/2\eta$, where η is the impedance of the medium.

Linearly Polarized Light

If one of the components vanishes ($a_x=0$, for example), the light is linearly polarized in the direction of the other component (the y direction). The wave is also linearly polarized if the phase difference $\varphi=0$ or π, since (6.1-4) gives $\mathcal{E}_y=\pm(a_y/a_x)\mathcal{E}_x$, which is the equation of a straight line of slope $\pm a_y/a_x$ (the + and − signs correspond to $\varphi=0$ or π, respectively). In these cases the elliptical cylinder in Fig. 6.1-1(b) collapses into a plane as illustrated in Fig. 6.1-2. The wave is therefore also said to have **planar polarization**. If $a_x=a_y$, for example, the plane of polarization makes an angle 45° with the x axis. If $a_x=0$, the plane of polarization is the y–z plane.

Circularly Polarized Light

If $\varphi=\pm\pi/2$ and $a_x=a_y=a_0$, (6.1-4) gives $\mathcal{E}_x=a_0\cos[2\pi\nu(t-z/c)+\varphi_x]$ and $\mathcal{E}_y=\mp a_0\sin[2\pi\nu(t-z/c)+\varphi_x]$, from which $\mathcal{E}_x^2+\mathcal{E}_y^2=a_0^2$, which is the equation of a circle. The elliptical cylinder in Fig. 6.1-1(b) becomes a circular cylinder and the wave is said to be circularly polarized. In the case $\varphi=+\pi/2$, the electric field at a fixed position z rotates in a clockwise direction when viewed from the direction toward which the wave is approaching. The light is then said to be **right circularly polarized**. The case $\varphi=-\pi/2$ corresponds to counterclockwise rotation and **left circularly**

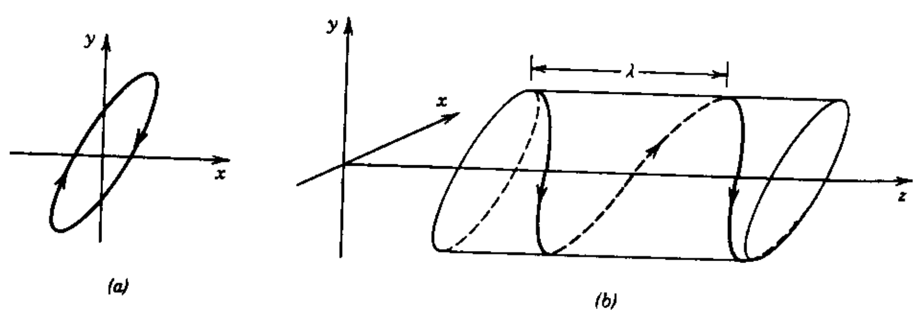


Figure 6.1-1 (a) Rotation of the endpoint of the electric-field vector in the x-y plane at a fixed position z. (b) Snapshot of the trajectory of the endpoint of the electric-field vector at a fixed time t.

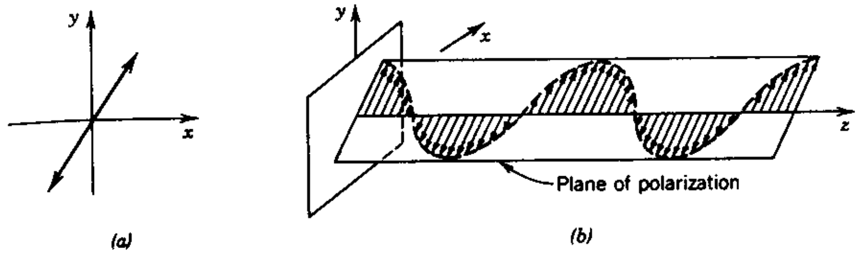


Figure 6.1-2 Linearly polarized light. (*a*) Time course at a fixed position z. (*b*) A snapshot (fixed time t).

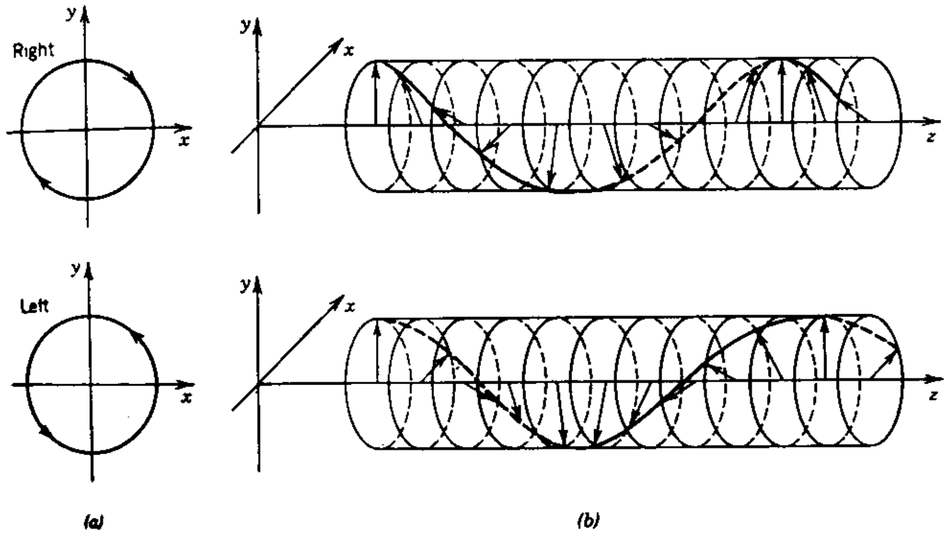


Figure 6.1-3 Trajectories of the endpoint of the electric-field vector of a circularly polarized plane wave. (*a*) Time course at a fixed position z. (*b*) A snapshot (fixed time t). The sense of rotation in (*a*) is opposite that in (*b*) because the traveling wave depends on $t - z/c$.

polarized light.† In the right circular case, a snapshot of the lines traced by the endpoints of the electric-field vectors at different positions is a right-handed helix (like a right-handed screw pointing in the direction of the wave), as illustrated in Fig. 6.1-3. For left circular polarization, a left-handed helix is followed.

B. Matrix Representation

The Jones Vector

A monochromatic plane wave of frequency ν traveling in the z direction is completely characterized by the complex envelopes $A_x = a_x \exp(j\varphi_x)$ and $A_y = a_y \exp(j\varphi_y)$ of the x and y components of the electric field. It is convenient to write these complex

†This convention is used in most textbooks of optics. The opposite designation is used in the engineering literature: in the case of right (left) circularly polarized light, the electric-field vector at a fixed position rotates counterclockwise (clockwise) when viewed from the direction toward which the wave is approaching.

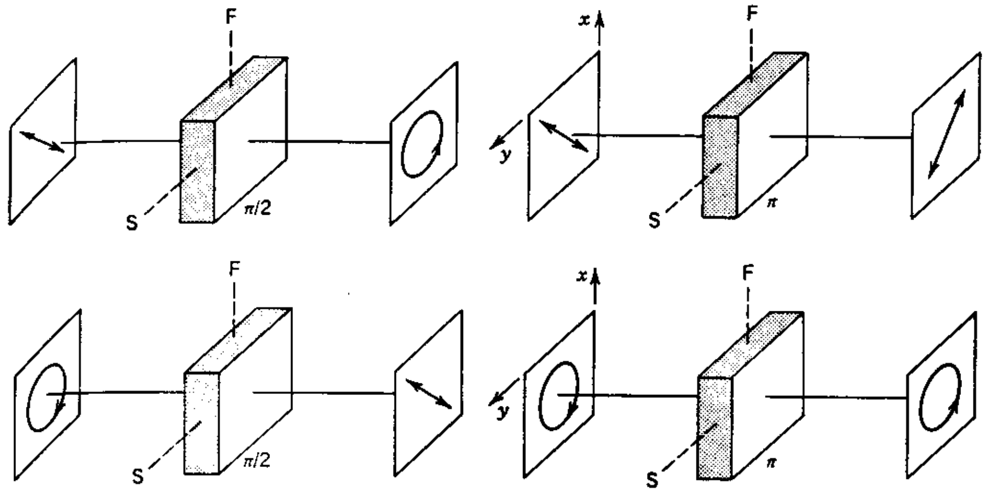


Figure 6.1-6 Operations of the quarter-wave ($\pi/2$) retarder and the half-wave (π) retarder. F and S represent the fast and slow axes of the retarder, respectively.

transforms a wave with field components (A_{1x}, A_{1y}) into another with components $(A_{1x}, e^{-j\Gamma}A_{1y})$, thus delaying the y component by a phase Γ, leaving the x component unchanged. It is therefore called a **wave retarder**. The x and y axes are called the fast and slow axes of the retarder, respectively. By simple application of matrix algebra, the following properties, illustrated in Fig. 6.1-6, may be shown:

- When $\Gamma = \pi/2$, the retarder (then called a **quarter-wave retarder**) converts linearly polarized light $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ into left circularly polarized light $\begin{bmatrix} 1 \\ -j \end{bmatrix}$, and converts right circularly polarized light $\begin{bmatrix} 1 \\ j \end{bmatrix}$ into linearly polarized light $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
- When $\Gamma = \pi$, the retarder (then called a **half-wave retarder**) converts linearly polarized light $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ into linearly polarized light $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, thus rotating the plane of polarization by 90°. The half-wave retarder converts right circularly polarized light $\begin{bmatrix} 1 \\ j \end{bmatrix}$ into left circularly polarized light $\begin{bmatrix} 1 \\ -j \end{bmatrix}$.

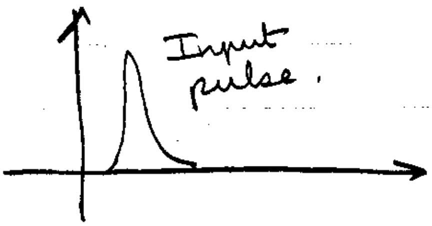
Polarization Rotators. The Jones matrix

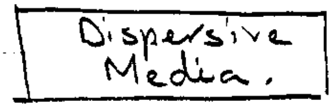
$$\boxed{\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}} \qquad (6.1\text{-}13)$$

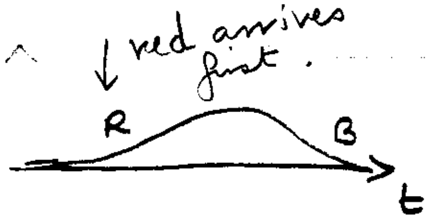
Polarization Rotator

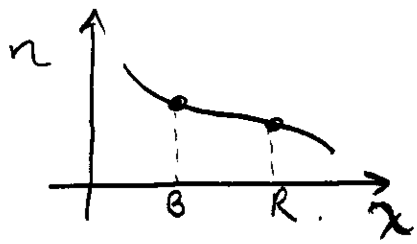
represents a device that converts a linearly polarized wave $\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix}$ into a linearly polarized wave $\begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{bmatrix}$ where $\theta_2 = \theta_1 + \theta$. It therefore rotates the plane of polarization of a linearly polarized wave by an angle θ. The device is called a **polarization rotator**.

Measures of dispersion – different frequencies travel at different speeds.





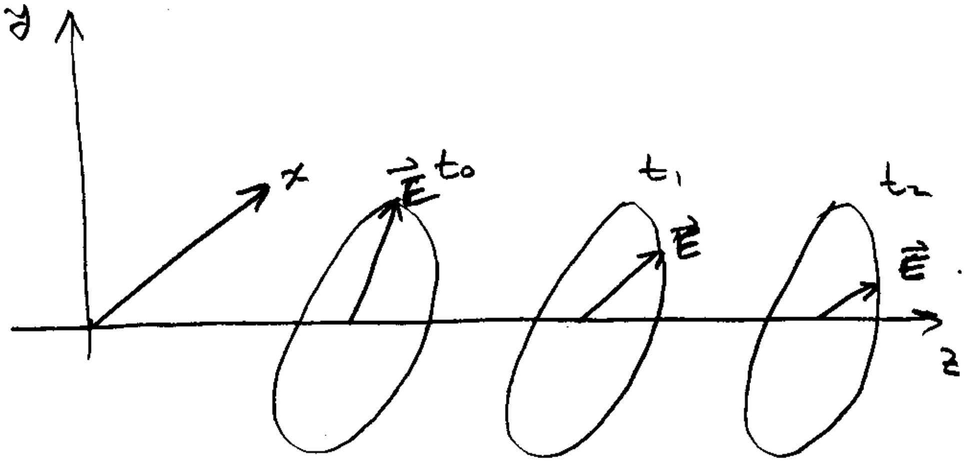




$$c = \frac{c_0}{n}.$$

Polarization of Light.

determined by the direction of the electric field vector $E(\vec{r},t)$.



Elliptically polarized.

Plane wave travelling in z-direction.

$$\vec{E}(z,t) = \mathrm{Re}\left\{\vec{A}\exp\left[j\,2\pi\nu\left(t-\frac{z}{c}\right)\right]\right\}.$$

where $\quad \vec{A} = A_x\hat{x} + A_y\hat{y} \quad$ A_x & A_y are complex vectors.

Polarizat Ellipse

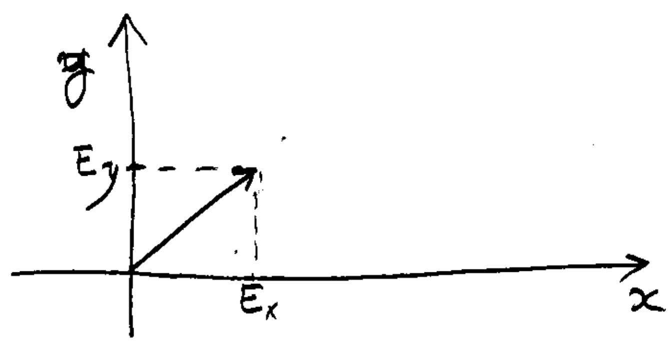
$$A_x = a_x \exp(j\phi_x) \quad A_y = a_y \exp(j\phi_y).$$

$$\Rightarrow \vec{E}(z,t) = E_x\,\hat{x} + E_y\,\hat{y}$$

$$E_x = a_x \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \phi_x\right]$$

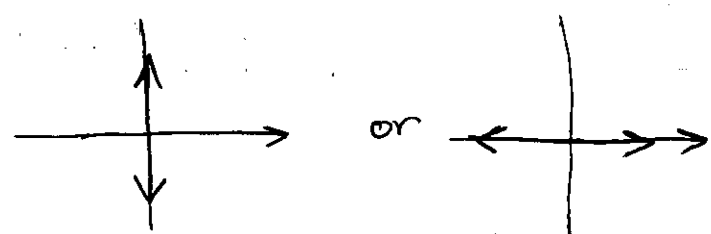
$$E_y = a_y \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \phi_y\right].$$

x, y components of $\vec{E}(z,t)$. E_x and E_y are periodic functions of $t - \frac{z}{c}$ oscillating at ν.



Linearly polarized

Suppose a_x or $a_y = 0$ $\Rightarrow$



time course at fixed z.

If $\phi_y - \phi_x = 0$ or π

$$\Rightarrow E_y = \pm (a_y/a_x)\, E_x$$

see handout. Fig 6.1-2.

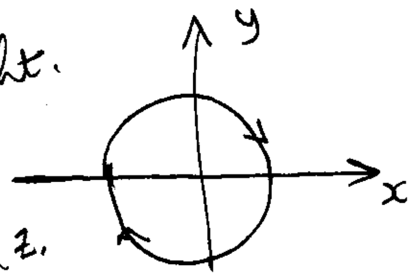
Circularly polarized light.

$\phi_y - \phi_x = \phi = \pm \frac{\pi}{2}$ & $a_x = a_y = a_0 \Rightarrow E_x = a_0 \cos\left(2\pi\nu\left(t - z/c\right) + \phi_x\right)$ and

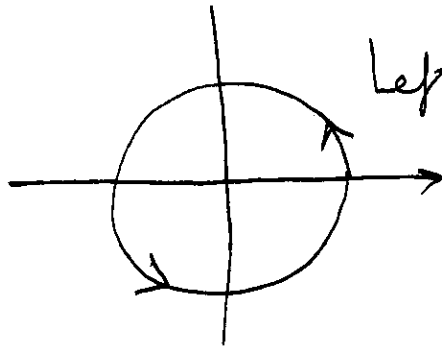
$$E_y = \mp a_0 \sin\left(2\pi\nu\left(t - \frac{z}{c}\right) + \phi_x\right). \Rightarrow E_x^2 + E_y^2 = a_0^2$$

$\Rightarrow$ circle

Right.



fixed z.



Left. circular.

Matrix Representation

$A_x = a_x \exp(j\phi_x)$, $A_y = a_y \exp(j\phi_y)$

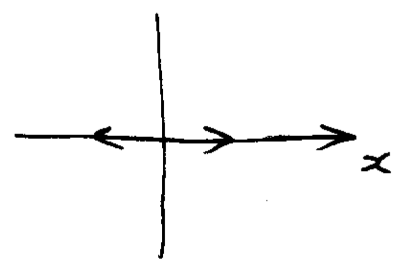
completely characterize the monochromatic wave.

We write $J = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$: Jones Vector.

$$I = \frac{\left(|A_x|^2 + |A_y|^2\right)}{2\eta} \qquad \underbrace{\frac{a_y}{a_x} = \frac{|A_y|}{|A_x|} \cdot \phi = \phi_y - \phi_x}_{\text{determine ellipticity}}$$

Examples.

Linear light in x-direction.



$$J = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$