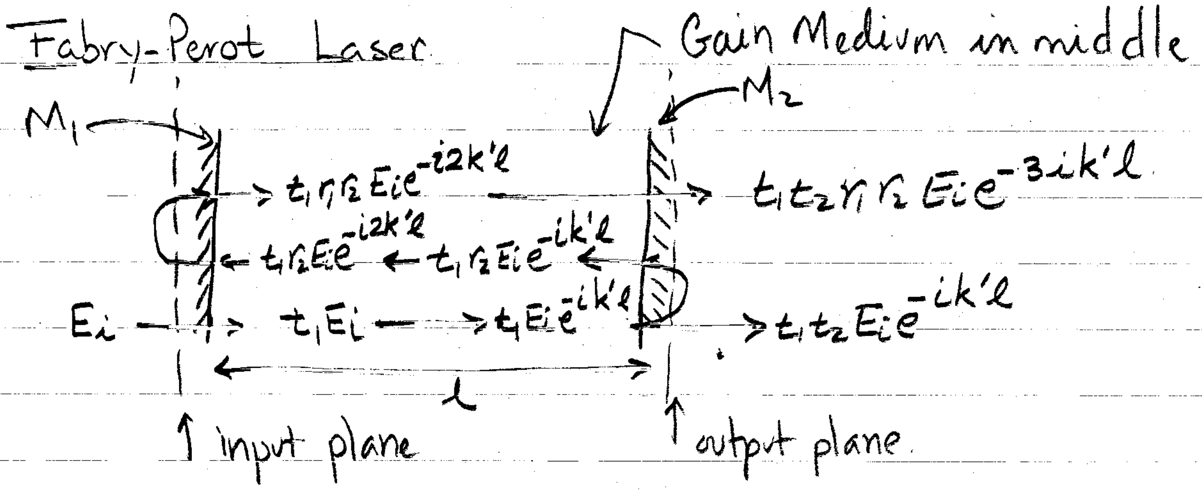
Theory of Laser Oscillation

Fabry-Perot Laser



where t_i: transmittance of Mirror i
r_i: reflectance of Mirror i
k': complex propagation constant
l: length of cavity

$$k'(\omega) = k + k\frac{\chi'(\omega)}{2n^2} - ik\frac{\chi''(\omega)}{2n^2} - i\frac{\alpha}{2}$$

when ω is far from laser transition $k'(\omega) = \underset{\text{delay}}{k} - i\underset{\text{absorption (loss)}}{\frac{\alpha}{2}}$

$\chi(\omega) = \chi'(\omega) - i\chi''(\omega)$ complex dielectric susceptibility

$$E_t = t_1 t_2 E_i e^{-ik'l}\Big[\underbrace{1 + r_1 r_2 e^{-i2k'l} + r_1^2 r_2^2 e^{-i4k'l} + \dots}_{\text{geometric series}}\Big]$$

$$= E_i\left[\frac{t_1 t_2 e^{-ik'l}}{1 - r_1 r_2 e^{-i2k'l}}\right]$$

$$E_t = E_i\left[\frac{t_1 t_2\, e^{-i(k+\Delta k)l}\, e^{(\gamma-\alpha)l/2}}{1 - r_1 r_2\, e^{-2i(k+\Delta k)l}\, e^{(\gamma-\alpha)l}}\right] \quad (1)$$

$$k' = k + \Delta k + i\frac{(\gamma - \alpha)}{2}$$

$$\Delta k = \frac{k\,\chi'(\omega)}{2n^2}$$

$$\gamma = -\frac{k\,\chi''(\omega)}{n^2} = (N_2 - N_1)\cdot\frac{\lambda^2}{8\pi n^2 t_{spont}}\,g(\nu).$$

Notice if $N_2 - N_1 > 0$ $\gamma > 0$ $\Rightarrow$ denominator of ① can become very small

$\Rightarrow$ E_t can become larger than E_i.

- Thus gain medium in Fabry perot makes for an amplifier $\left|\frac{E_t}{E_i}\right|^2$ - Power Gain.

If the denominator of ① becomes 0 $\frac{E_t}{E_i} \to \infty$.

$$r_1 r_2 e^{-2i(k+\Delta k)l}\, e^{(\gamma(\omega) - \alpha)l} = 1.$$

$\boxed{\text{or } E_i = 0 \text{ still gives out light}}$

This is true when wave inside starting plane is the same value after 1 round trip. (same amplitude and same phase.)

Separate equation into amplitude and phase requirements.

$$\Rightarrow r_1 r_2 e^{(\gamma_t(\omega) - \alpha)l} = 1. \quad (\text{amplitude})$$

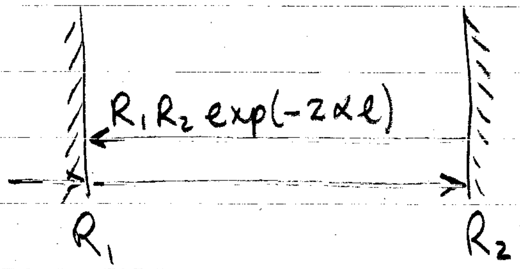
Threshold gain when $\gamma_t(\omega) = \alpha - \frac{1}{l}\ln r_1 r_2$

Phase condition $2[k + \Delta k(\omega)]\, l = 2\pi m \qquad m = 1, 2, 3, \ldots$

From gain: $N_t \equiv (N_2 - N_1)_t = \dfrac{8\pi n^2 t_{spont}}{g(\nu)\, \lambda^2}\left(\alpha - \dfrac{1}{l}\ln r_1 r_2\right).$

N_t : Population inversion density at threshold.

Photon Lifetime



Fractional intensity loss is: $1 - R_1 R_2 \exp(-2\alpha l)$.

One Round trip time: $\dfrac{2 l n}{c}$

$\Rightarrow$ Exponential decay with a time constant τ_{ph}

$$\frac{1}{\tau_{ph}} = \frac{(1 - R_1 R_2 e^{-2\alpha l})\, c}{2 l n}$$

$$\frac{d\mathcal{E}}{dt} = -\frac{\mathcal{E}}{\tau_{ph}} \qquad \mathcal{E} : \text{energy store.}$$

When $R_1 R_2 e^{-2\alpha l} \simeq 1$ then $1 - x \simeq -\ln x,\ x \approx 1$

$$\Rightarrow \frac{1}{\tau_{ph}} \simeq \frac{c}{n}\left[\alpha - \frac{1}{l}\ln(r_1 r_2)\right].$$

$$N_t \equiv (N_2 - N_1)_t = \frac{8\pi n^3 \nu^2 \tau_{spont}}{c^3 \tau_{ph}\, g(\nu)}.$$

(See Figure 6-2).

Oscillation Frequency.

$$kl\left[1+\frac{\chi'(\nu)}{2n^2}\right] = m\pi$$

$$\nu_m = \frac{mc}{2ln.}$$ (mth resonance frequency of passive cavity).

$$\chi'(\nu) = \frac{2(\nu_0-\nu)}{\Delta\nu}\chi''(\nu).$$

$$\gamma(\nu) = -\frac{k\,\chi''(\nu)}{n^2}$$

$$\Rightarrow \nu\left[1-\frac{(\nu_0-\nu)}{\Delta\nu}\frac{\gamma(\nu)}{k}\right] = \nu_m.$$

ν_0: center frequency of lineshape function.

if ν_m is near ν_0, ν will be close to ν_m.

$$\nu \simeq \nu_0$$

$$\Rightarrow \nu = \nu_m - (\nu_m-\nu_0)\frac{\gamma(\nu_m)\,c}{2\pi n\Delta\nu}.$$

Recall that oscillation condition: $r_1 r_2\, e^{(\gamma_t(\omega)-\alpha)l} = 1.$

let: $r_1 = r_2 = \sqrt{R}$, $\alpha=0$,

if $R = 1-\Delta$ $\Rightarrow \sqrt{R}\,e^{\gamma_t l} = 1$, $\Delta \ll 1.$

or

$$(1-\Delta)\,e^{\gamma_t l} = 1$$

small $\gamma_t l$ $\quad (1-\Delta)(1+\gamma_t l) = 1 \quad (1+\gamma_t l) \simeq 1+\Delta$

$$\gamma_t \simeq \frac{\Delta}{l} \simeq \left(\frac{1-R}{l}\right)$$

So $\gamma_t \simeq \dfrac{(1-R)}{l}$.

$$\Delta\nu_{\frac{1}{2}} \simeq \frac{c(1-R)}{2\pi n l}$$

- Passive Cavity: $\Delta\nu_{\frac{1}{2}} = \dfrac{c}{2\pi n l}$.

$$\therefore\ \nu = \nu_m - (\nu_m - \nu_0)\frac{\Delta\nu_{\frac{1}{2}}}{\Delta\nu}.$$

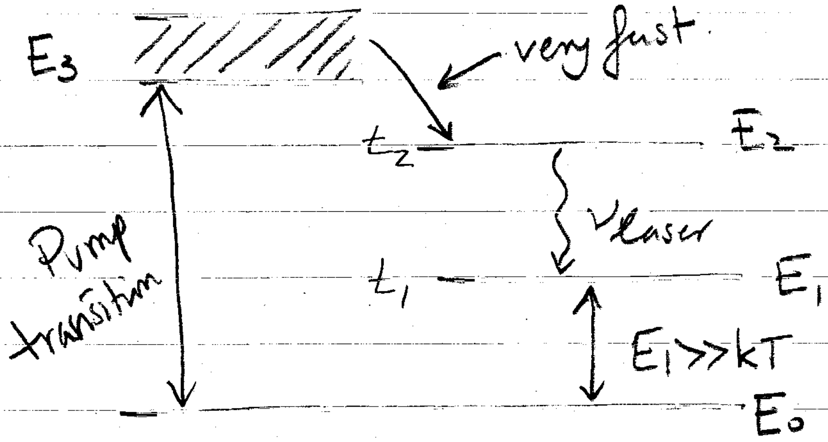
if $\nu_m = \nu_0$ i.e., line center is perfectly matched with resonator frequency.

then $\nu = \nu_m$ - laser will operate at line center.

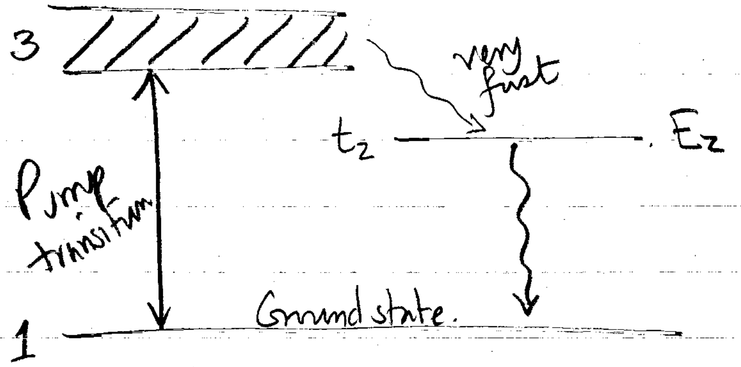
However, if $\nu_m \neq \nu_0$
laser operates with a slightly shifted frequency pulled slightly towards ν_0

$\Rightarrow$ frequency pulling. (see figure 6-3).

Three and Four level lasers.



If t_1 very short $\Rightarrow N_t \simeq N_2$



For lasing we need

$$N_2 = \frac{N_0}{2} + \frac{N_t}{2}$$

$$N_1 = \frac{N_0}{2} - \frac{N_t}{2}.$$

$$N_2 - N_1 = N_t.$$

For most lasers $N_0 >> N_t$

$$\Rightarrow \frac{(N_2)_{3\text{-level}}}{(N_2)_{4\text{-level}}} \sim \frac{N_0}{2N_t}.$$

For $\frac{N_0}{2}$ atoms in the upper state $\Rightarrow$

$$(P_s)_{3\text{-level}} = \frac{N_0 h\nu V}{2t_2} \qquad [V\text{ - volume}]$$

$$(P_s)_{4\text{-level}} = \frac{N_t h\nu V}{t_2} \qquad \left(\text{from } \frac{dN}{dt} = \frac{-N}{\tau}\right)$$

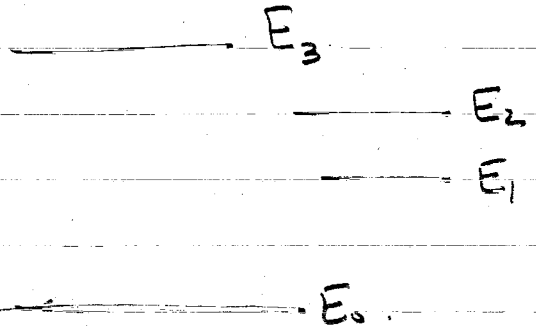
P_s – critical fluorescence power.

$$(P_s)_{4\text{-level}} = \frac{8\pi n^3 h \Delta\nu V}{\lambda^3 t_c} \frac{t_{spont}}{t_2}$$

$$\Delta\nu \cong \frac{1}{g(\nu_0)}$$

Power in Laser Oscillators.

Rate Equations.



$$\frac{dN_2}{dt} = -N_2 w_{21} - W_i (N_2 - N_1) + R_2$$

$$\frac{dN_1}{dt} = -N_1 w_{10} + N_2 w_{21} + W_i (N_2 - N_1) + R_1$$

$w_{ij} = \frac{1}{\tau_{ij}} = A_{ij}$ $\quad$ W_i: induced transition rate.

(assumed homogeneous system
- inhomogeneous system $W_i(\nu)$ must be used).

Steady state $\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$.

$$N_2 - N_1 = \frac{R_2 \left[1 - (w_{21}/w_{10})(1 + R_1/R_2) \right]}{W_i + w_{21}}$$

For $N_2 - N_1 > 0 \Rightarrow \frac{w_{21}}{w_{10}} < 1 \Rightarrow w_{21} < w_{10}$

Effective pumping rate $R = R_2 \left[1 - \frac{w_{21}}{w_{10}} \left(1 + \frac{R_1}{R_2} \right) \right]$

$$N_2 - N_1 = \frac{R}{W_i + W_{21}}$$

Below threshold $W_i = 0$. $\Rightarrow N_2 - N_1 \propto R$.

This is true until $N_2 - N_1 = N_t$ or $\boxed{R = N_t W_{21}}$.

Gain a ν_0 is large enough to make up for cavity loss.

Further increases in $N_2 - N_1$ are impossible in steady-state condition ~~since~~. $W_i \uparrow \Rightarrow$ Energy $\uparrow \Rightarrow$ violates steady state.

Under steady state $\quad N_t = \dfrac{R}{W_i + W_{21}} \qquad W_i = \dfrac{R}{N_t} - W_{21} \quad R \geq N_t W_{21}$

Power generated by stimulated emission.

$$P_e = N_t V W_i h\nu.$$

$$\frac{P_e}{Vh\nu} = N_t W_{21}\left(\frac{R}{N_t W_{21}} - 1\right) \qquad R \geq N_t W_{21}$$

$$\frac{P_e}{Vh\nu} = N_t W_{21}\left(\frac{R}{P/t_c} - 1\right) \qquad R \geq \frac{P}{t_c} \qquad P = \frac{8\pi n^3 \nu^2}{c^3 g(\nu_0)}$$

or $P_e = P_s\left(\dfrac{R}{R_t} - 1\right)$ — emitted power

R_t — threshold pump power, R pumping rate.